Performance Based Learning and Assessment Task

Triangular Irrigation

I. ASSESSSMENT TASK OVERVIEW & PURPOSE:

In this performance task, students will use reflections and proving triangles congruent to supply water from a river to a farmer's corn and tomatoes. They need to install one water pump, and install two pipes that will run from the water pump to the crops. It is the students' job to find the best location along the river that will minimize the amount of piping needed.

II. UNIT AUTHOR:

Tiffany Sakaguchi, Cave Spring Middle School, Roanoke County Schools

III. COURSE:

Geometry

IV. CONTENT STRAND:

Geometry, Measurement

V. OBJECTIVES:

Students will be able to: 1)Predict two different locations to install the water pump. Find the distance to piping needed using the distance formula and Pythagorean Theorem, 2)Compare total distances with other students. Make a conjecture about the *best* location for the water pump, 3)Relocate crops to make corn, tomatoes, and water pump collinear. Make observations about the optimum point for the water pump and calculate piping needed, 4)Use reflections to locate the tomatoes on the opposite side of the river. Since the shortest distance between two objects is a straight line, students will use the reflection and congruent triangles to determine where to install the water pump, 5)Construct a line from the corn to the reflected image of the tomatoes. Find where this intersection intersects the river, and prove this is the best location for the water pump using congruent triangles.

VI. REFERENCE/RESOURCE MATERIALS:

Students will use: Classroom set of graphing calculators, Classroom set of worksheet, Straight edge, GeoGebra applet for optional teacher/student exploration

VII. PRIMARY ASSESSMENT STRATEGIES:

The task includes an assessment component that performs two functions: (1) for the student it will be a checklist and provide a self-assessment and (2) for the teacher it will be used as a rubric. Students will use the distance formula, Pythagorean Theorem, reflections, and proving triangles congruent to correctly determine the best location of a water pump to minimize the distance of piping needed to get water to the corn and tomatoes.

VIII. EVALUATION CRITERIA:

Students will follow a guided activity worksheet and will be graded based upon the attached rubric.

IX. INSTRUCTIONAL TIME:

60 minutes

Triangular Irrigation

Strand

Geometry, Measurement

Mathematical Objective(s)

Students will be able to: 1)Predict two different locations to install the water pump. Find the distance to piping needed using the distance formula and Pythagorean Theorem, 2)Compare total distances with other students. Make a conjecture about the *best* location for the water pump, 3)Relocate crops to make corn, tomatoes, and water pump collinear. Make observations about the optimum point for the water pump and calculate piping needed, 4)Use reflections to locate the tomatoes on the opposite side of the river. Since the shortest distance between two objects is a straight line, students will use the reflection and congruent triangles to determine where to install the water pump, 5)Construct a line from the corn to the reflected image of the tomatoes. Find where this intersection intersects the river, and prove this is the best location for the water pump using congruent triangles.

Related SOL

- G.3a (investigate and using formulas for finding distance, midpoint, and slope)
- G.3d (determine whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods)
- G.6 (student, given information in the form of a figure or statement, will prove two triangles are congruent, using algebraic and coordinate methods as well as deductive proofs)

NCTM Standards

- Establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others
- Investigate conjectures and solve problems involving two- and three- dimensional objects represented with Cartesian coordinates
- Understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, function notation, and matrices
- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Materials/Resources

Students will need: Classroom set of graphing calculators, Classroom set of worksheets, Straight Edge, GeoGebra applet for optional teacher/student exploration

Assumption of Prior Knowledge

Students should be familiar with:

- Calculating distances using the distance formula and Pythagorean Theorem
- Reflections across the axes and $y=x$.
- Proofs for congruent triangles

Introduction: Setting Up the Mathematical Task

- Teachers should explain that the students are going to be determining the best location for a farmer to put a water pump along a river in order to reduce the amount of piping needed to water his corn and tomatoes.
- The activity will be divided into three parts. There should be group discussion between parts 2 and 3.

Student Exploration

Student/Teacher Actions (Part 1)

- Students should begin by finding two possible locations for the farmer to place the water pump (question #3). They will then proceed to find the total distance of piping need from both locations using the distance formula and Pythagorean Theorem (questions #4 and #5).
- It may be beneficial to draw an example on the board of one possible location and show how there will be two pipes from the pump, one going to the corn and the other going to the tomatoes.
- Stress to the students that they do not have to use integer coordinates for the location of the pump. For higher level students, teachers may want to make their students choose at least one location for question #3 that is not at an integer coordinate location. This will make the calculations a bit more complex for distance.
- After students have made their calculations, they should share their findings with other students in the class.
- Students should start developing an idea on where the "best" location for the water pump would be.
- Ask students if they notice any patterns or trends? Do they notice that the distances become smaller when they approach a certain point and them larger after a particular point? They should notice an optimal range for the water pump.

Monitoring Student Responses (Part 1)

- The teacher will assist students by answering questions and providing suggestions along the way.
- Students should communicate their thinking verbally and in written form when sharing their distances with the class.
- After students have finished question #7, discuss as a class their findings. Provide feedback and answer any questions.

- Determine if students believe there is one or more "best" locations for the water pump.

Small Group Work (Part 2)

Students will continue Part 3 by working in small groups of 2 or 3.

Student/Teacher Actions (Part 2)

- Student will start by exploring what happens if the corn, tomatoes, and water pump are collinear in questions #6-9. They will reflect the corn and tomatoes across the x-axis and find the total distance of piping.
- Student should notice that the location of *P** is the same, no matter whether they reflect the corn or tomatoes.
- After students complete questions #6-9, the teacher may want to have class discussion about why they have just reflected the corn and tomatoes across the river. Teachers should ask questions to lead students to the idea that the shortest distance between two points is a straight line.
- Teachers should ask if there is any significance to where the new line crosses the *x*-axis. Students should eventually come to the conclusion that the intersection is the best location for the water pump to minimize the amount of piping.
- Once students have determined the best location for the water pump, they will need to use Algebra to determine the equation of the line and find the *x*-intercept. They may need a review of these skills.
- Ask students how the distance from *P** to *T'* relates to the distance from *P** to *T*. Students should conclude that these two distances are the exact same.
- Students will need to prove that *TP** and *T'P** are congruent by first proving triangles *QTP** and *QT'P** are congruent.

Monitoring Student Responses (Part 2)

- The teacher will assist students by answering questions and providing suggestions along the way.
- Students should communicate their thinking verbally and in written form when sharing their distances with the class.
- As a possible extension, teachers can use the following GeoGebra applet (http://tube.geogebra.org/student/m128250) to model the possible locations of the water pump. This can be done by the teacher in front of the classroom, or in groups when computers are accessible.
- Using the GeoGebra applet can lead to discussion on what would have happened if the farmer did not choose the *best* location. Students may observe that although the best location of 5.333 on the *x*-axis gives the least amount of piping, there are other good locations nearby that do not require much more piping.
- Possible question: In what range along the river is a good location for the pump?
- Possible question: Can you think of any reasons why the farmer may not be able to put the pump at 5.333 along the river? What would you suggest the farmer to do?

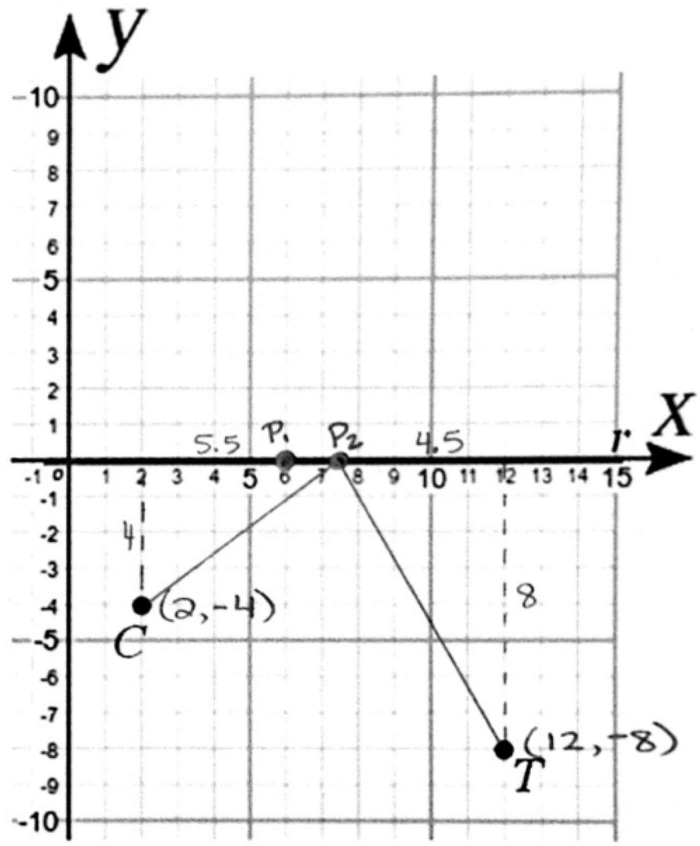
Assessment List and Benchmarks

Class worksheets and assessment rubrics are attached.

Triangular Irrigation

PART 1: Approximating a Water Pump

A farmer wants to run two pipes from river *r* to his corn *C* (2, –4) and tomatoes *T* (12, –8). At what point along the river should the water pump be installed to minimize the amount of piping needed to supply water to the corn and tomatoes?



1. Choose two points along the river that you think would be a good location for the water pump. Label these points P_1 and P_2. (Note: These points do not have to be integer points.)

   $P_1(6,0)$ $P_2(7.5,0)$

2. Find the total length of piping that would be needed if the water pump is located at P_1. Calculate $CP_1 + TP_1$ using the **distance formula**.

$CP_1 = \sqrt{(2-6)^2+(-4-0)^2}$
$= \sqrt{(-4)^2+(-4)^2}$
$= \sqrt{16+16}$
≈ 5.66

$TP_1 = \sqrt{(12-6)^2+(-8-0)^2}$
$= \sqrt{(6)^2+(-8)^2}$
$= \sqrt{36+64}$
$= 10$

$CP_1 + TP_1 = 5.66 + 10 = 15.66$

3. Find the total length of piping that would be needed if the water pump is located at P_2. Calculate $CP_2 + TP_2$ using the **Pythagorean Theorem**.

$4^2 + 5.5^2 = c^2$
$46.25 = c^2$
$c \approx 6.8$

$4.5^2 + 8^2 = c^2$
$84.25 = c^2$
$c \approx 9.18$

$CP_2 \approx 6.8$ $TP_2 = 9.18$

$CP_2 + TP_2 = 6.8 + 9.18 = 15.98$

4. Does P_1 or P_2 give you the least amount of piping?

   P_1 gave me the least amount of piping.

5. Compare your total distances with your classmates. Write a brief description of your observations. Include your hypothesis on the "best" location for *P* that minimizes the total amount of piping and why you believe this is true. How much piping is needed for this *best* location?

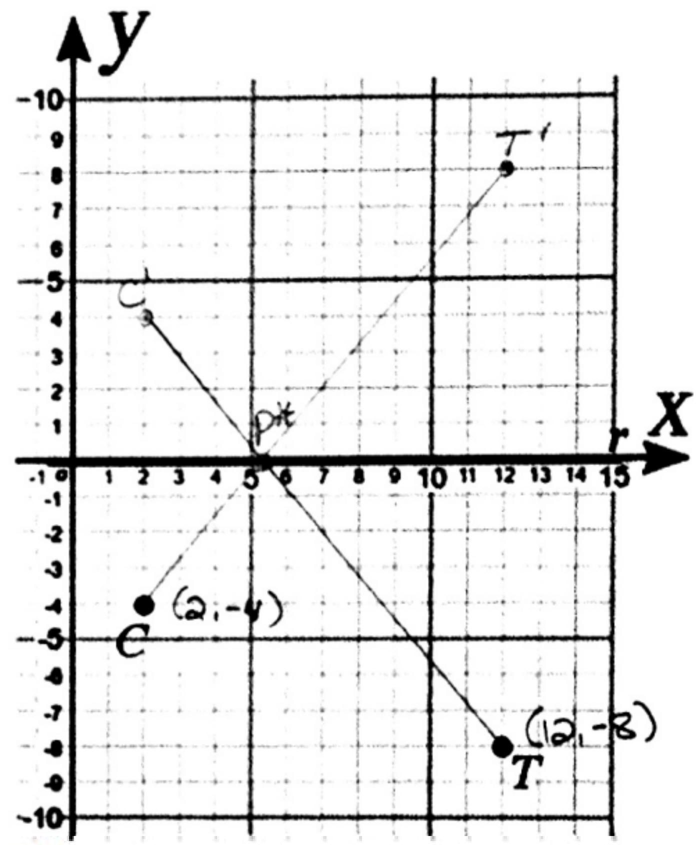
   It seems as if the best location for the water pump is somewhere between 5 and 6. As you move further from this range, the total distance will increase. I believe the best location will be at 5.5. The distance of the piping would be 15.63.

$CP = \sqrt{(2-5.5)^2+(-4-0)^2}$
$= \sqrt{(-3.5)^2+(-4)^2}$
$= \sqrt{12.25+16}$
≈ 5.32

$PT = \sqrt{(12-5.5)^2+(-8-0)^2}$
$= \sqrt{(6.5)^2+(-8)^2}$
$= \sqrt{42.25+64}$
≈ 10.31

PART 2: Determine the Best Location for the Water Pump

Let *P** be the optimal location of the pump. If the farmer had an option to choose where the river was running, he would have the river run between points *C* and *T*, with *P** located on the line segment *CT*. Because the river is not located between *C* and *T*, we must find the BEST location that minimizes the amount of piping. There are two optimal choices found by making *C, P**, and *T* collinear.



6. How can we change the location of *C* (or *T*) so that *C, P**, and *T* are collinear?

We could move the corn or tomatoes to the other side of the river making the points collinear.

7. What would happen if the corn was planted at (2, 4)? What would be the optimal location for the water pump, and what would be the length of the piping needed?

If the corn was at (2,4), all pts would be collinear. P* would be between 5 & 6, closer to 5.

$\sqrt{(12-2)^2+(-8-4)^2}$ Piping Needed: 15.62

≈ 15.62

8. What would happen if the tomatoes were planted at (12, 8)? What would be the optimal location for the water pump, and what would be the length of the piping needed?

If the tomatoes were at (12,8), all pts would be collinear.
P* is between 5 & 6, closer to 5.

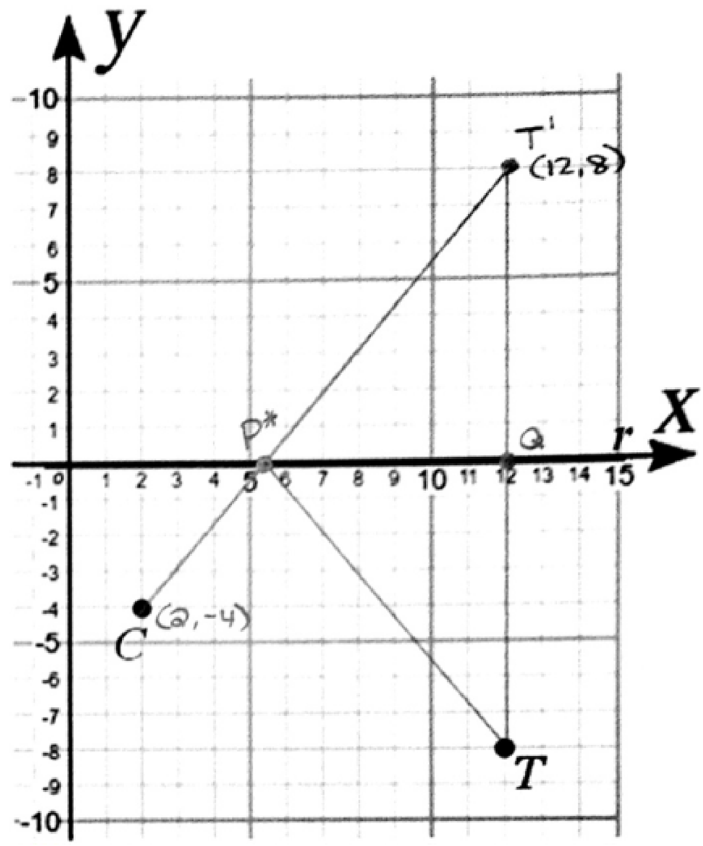
$\sqrt{(12-2)^2+(8-(-4))}$ Piping Needed: 15.62

≈ 15.62

9. What do you notice about the location of *P** in questions 5, 7, and 8? How would you change your answer to question 5 based on what you found in questions 7 and 8?

I notice my original guess of 5.5 was very close. I would like to change my answer to 5.25 so that P* is a little closer to 5 like I noticed in #7 & #8.

10. An engineer suggested the following strategy: Reflect point *T* across the river (*x*-axis) and label this point *T′*.



11. Find the distance between *C* and *T′* using the method of your choice. How does this total distance compare with the total distances from questions 7 and 8?

$CT' = \sqrt{(12-2)^2 + (8-(-4))^2}$
$= \sqrt{(10)^2 + (12)^2}$
$= \sqrt{100+144}$
≈ 15.62

12. Label the point where line *CT* crosses the *x*-axis *P**. Draw line *TT′*. Label the point where this line crosses the *x*-axis *Q*. Create triangles *QTP** and *QT′P**.

13. Write an equation for the line through points *C* and *T′*. Use this equation to solve for the *x*-intercept of the line.

Slope $= \frac{8-(-4)}{12-2} = \frac{12}{10} = \frac{6}{5}$ or 1.2

$y+4 = 1.2(x-2)$
$y+4 = 1.2x - 2.4$
$y = 1.2x - 6.4$

Solve for x-intercept, plug in y = 0
$0 = 1.2x - 6.4$
$6.4 = 1.2x$
$x = 5.\overline{3}$ or $5\frac{1}{3}$

14. Given $QT \cong QT'$ and $\angle P^*QT$ and $\angle P^*QT'$ are right angles, prove $TP^* \cong T'P^*$.

Statement	Reason
$QT \cong QT'$	Given
$\angle P^*QT$ and $\angle P^*QT'$ are right angles	Given
$\angle P^*QT \cong \angle P^*QT'$	All right angles congruent
$P^*Q \cong P^*Q$	Reflexive Prop of congruence
$\triangle P^*QT \cong \triangle P^*QT'$	SAS
$TP^* \cong T'P^*$	CPCTC

15. Explain how you know that *P** is the point along the river that best minimizes the amount of piping needed?

The shortest distance between 2 pts is a straight line. Since $TP^* \cong T'P^*$, the best place to put the water pump would be at $5\frac{1}{3}$ along the river (x-axis). The total distance is 15.62.

16. Is the suggestion by the engineer the only way to find the optimal solution? Why or why not?

No. The suggestion by the engineer is not the only way to find the optimal solution. He could have reflected the corn instead of the tomatoes and got the same solution.

- Students will be graded with the following rubric:

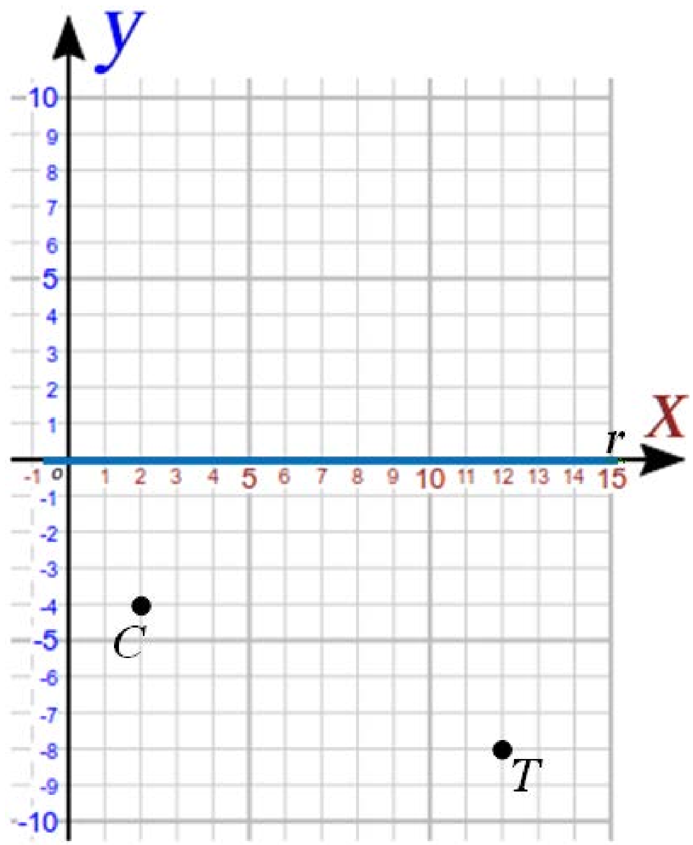
	3	2	1	0
#1: Two points chosen along river and labeled	Complete and accurate.	Partially complete and mostly accurate.	Attempted completion, but incorrect.	Not complete.
#2: Length calculated using the distance formula	All calculations shown, complete, and accurate.	Partial calculations are shown, complete, and accurate.	Attempted to complete calculations but incorrect.	Not complete.
#3: Length calculated using the Pythagorean Theorem	All calculations shown, complete, and accurate.	Partial calculations are shown, complete, and accurate.	Attempted to complete calculations but incorrect.	Not complete.
#4: Determine whether P_1 or P_2 uses least amount of piping.	Correct.	Partially correct.	Incorrect, work shown.	Not complete.
#5: Compare and describe observations. Calculate needed piping.	Used precise mathematical language to clearly communicate thinking. All calculations shown, complete, and accurate.	Partially communicated thinking and explanation. Partial calculations are shown, complete, and accurate.	Used minimal communication and explanation. Attempted to complete calculations but incorrect.	Not complete.
#6: How to make C, T, and P* collinear.	Correct.	Partially correct.	Incorrect, work shown.	Not complete.
#7: Relocate corn. Description and calculation of piping.	Used precise mathematical language to clearly communicate thinking. All calculations shown, complete, and accurate.	Partially communicated thinking and explanation. Partial calculations are shown, complete, and accurate.	Used minimal communication and explanation. Attempted to complete calculations but incorrect.	Not complete.
#8: Relocate tomatoes. Description and calculation of piping.	Used precise mathematical language to clearly communicate thinking. All calculations shown, complete, and accurate.	Partially communicated thinking and explanation. Partial calculations are shown, complete, and accurate.	Used minimal communication and explanation. Attempted to complete calculations but incorrect.	Not complete.
#9: Observations of P*.	Used precise mathematical language to clearly communicate thinking.	Partially communicated thinking and explanation.	Used minimal communication and explanation.	Not complete.
#10: Reflect point T across *x*-axis	Complete and accurate.	Partially complete and mostly accurate.	Attempted completion, but incorrect.	Not complete.

#11: Distance calculated using any method	All calculations shown, complete, and accurate.	Partial calculations are shown, complete, and accurate.	Attempted to complete calculations but incorrect.	Not complete.
#12: Line drawn and labeled, triangles drawn	Complete and accurate.	Partially complete and mostly accurate.	Attempted completion, but incorrect.	Not complete.
#13: Equation for line	All calculations shown, complete, and accurate.	Partial calculations are shown, complete, and accurate.	Attempted to complete calculations but incorrect.	Not complete.
#13: Calculation for *x*-intercept	All calculations shown, complete, and accurate.	Partial calculations are shown, complete, and accurate.	Attempted to complete calculations but incorrect.	Not complete.
#14: Triangle proof	Complete and accurate.	Partially complete and mostly accurate.	Attempted completion, but incorrect.	Not complete.
#15: Explanation and reasoning	Used precise mathematical language to clearly communicate thinking.	Partially communicated thinking and explanation.	Used minimal communication and explanation.	Not complete.
#16: Is the engineers suggestions the correct solution?	Used precise mathematical language to clearly communicate thinking.	Partially communicated thinking and explanation.	Used minimal communication and explanation.	Not complete.

Triangular Irrigation

PART 1: Approximating a Water Pump

A farmer wants to run two pipes from river *r* to his corn *C* (2, –4) and tomatoes *T* (12, –8). At what point along the river should the water pump be installed to minimize the amount of piping needed to supply water to the corn and tomatoes?



1. Choose two points along the river that you think would be a good location for the water pump. Label these points P_1 and P_2. (Note: These points do not have to be integer points.)

2. Find the total length of piping that would be needed if the water pump is located at P_1. Calculate $CP_1 + TP_1$ using the **distance formula**.

3. Find the total length of piping that would be needed if the water pump is located at P_2. Calculate $CP_2 + TP_2$ using the **Pythagorean Theorem**.

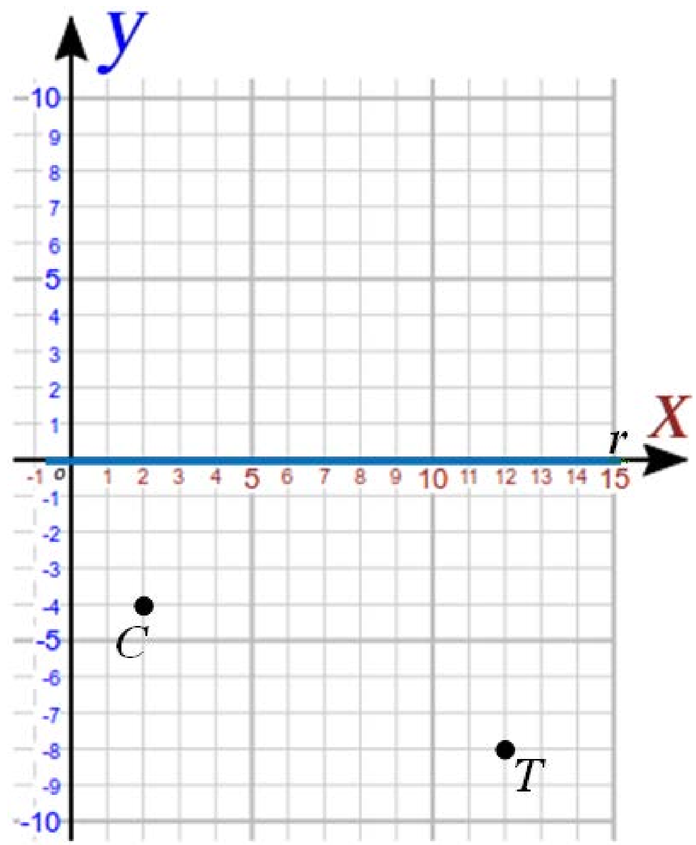
4. Does P_1 or P_2 give you the least amount of piping?

5. Compare your total distances with your classmates. Write a brief description of your observations. Include your hypothesis on the "best" location for *P* that minimizes the total amount of piping and why you believe this is true. How much piping is needed for this *best* location?

PART 2: Determine the Best Location for the Water Pump

Let *P** be the optimal location of the pump. If the farmer had an option to choose where the river was running, he would have the river run between points *C* and *T*, with *P** located on the line segment *CT*. Because the river is not located between *C* and *T*, we must find the BEST location that minimizes the amount of piping. There are two optimal choices found by making *C, P*,* and *T* collinear.

6. How can we change the location of *C* (or *T*) so that *C, P*,* and *T* are collinear?



7. What would happen if the corn was planted at (2, 4)? What would be the optimal location for the water pump, and what would be the length of the piping needed?

8. What would happen if the tomatoes were planted at (12, 8)? What would be the optimal location for the water pump, and what would be the length of the piping needed?

9. What do you notice about the location of *P** in questions 5, 7, and 8? How would you change your answer to question 5 based on what you found in questions 7 and 8?

10. An engineer suggested the following strategy: Reflect point *T* across the river (*x*-axis) and label this point *T'*.

11. Find the distance between *C* and *T'* using the method of your choice. How does this total distance compare with the total distances from questions 7 and 8?

12. Label the point where line *CT* crosses the *x*-axis *P**. Draw line *TT'*. Label the point where this line crosses the *x*-axis *Q*. Create triangles *QTP** and *QT'P**.

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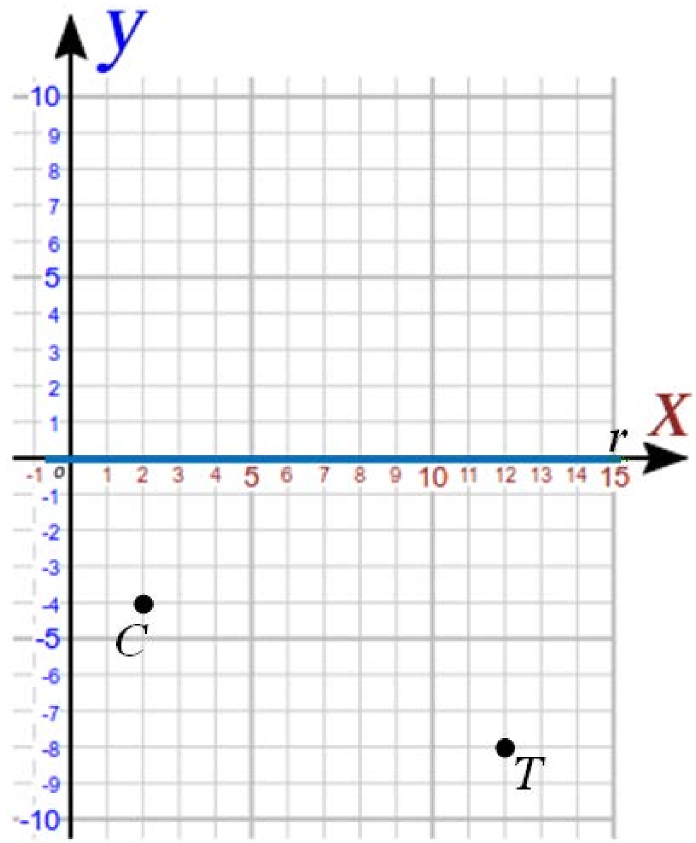

14. Given $QT \cong QT'$ and $\angle P^*QT$ and $\angle P^*QT'$ are right angles, prove $TP^* \cong T'P^*$.

Statement	Reason

15. Explain how you know that *P** is the point along the river that best minimizes the amount of piping needed?

16. Is the suggestion by the engineer the only way to find the optimal solution? Why or why not?